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Conservative Matrices in Summability of Series

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Abstract. Das [3] introduced the class of absolute kth-power conservative matrices for $k \geq 1$, denoted by $B(A_k)$. In the present paper, we generalize the class $B(A_k)$ to a general one named $B(\alpha_n, \beta_n; \gamma_n, \delta_n; \varphi)$ and give some sufficient conditions for a matrix belongs to the new class $B(\alpha_n, \beta_n; \gamma_n, \delta_n; \varphi)$ when φ is convex. As applications of the general result, we investigate the conservatives of Cesáro matrices and Riesz matrices.

1. Introduction

Let $\{s_n\}$ be the partial sums of the infinite series $\sum_{n=0}^{\infty} a_n$, The Cesáro means of order α of the series $\sum_{n=0}^{\infty} a_n$ are defined by

$$\sigma_n^\alpha := \frac{1}{A_n^\alpha} \sum_{j=0}^{n} A_{n-j}^{\alpha-1} s_j, \ n = 0, 1, \cdots,$$

where

$$A_n^\alpha := \frac{\Gamma(n+\alpha+1)}{\Gamma(\alpha+1)\Gamma(n+1)}, \ n = 0, 1, \cdots.$$

Let (C, α) be the Cesáro matrix of order α, that is, (C, α) be the lower triangular matrix $\left(A_{n-v}^{\alpha-1}/A_n^\alpha\right)$.

Flett [4] introduced the concept of absolute summability of order k. A series $\sum_{n=0}^{\infty} a_n$ is summable $|C, \alpha|_k$, $k \geq 1$, $\alpha > -1$, if

$$\sum_{n=0}^{\infty} n^{k-1} \left|\sigma_{n-1}^\alpha - \sigma_n^\alpha\right|^k < \infty.$$

In 1970, Das [3] defined the so-called absolutely kth-power conservative matrix as follows: A matrix $T := \left(t_{nj}\right)$ to be absolutely kth-power conservative for $k \geq 1$, denoted by $T \in B(A_k)$, that is, if $\{s_n\}$ satisfies

$$\sum_{n=1}^{\infty} n^{k-1} |s_n - s_{n-1}|^k < \infty,$$

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then

$$\sum_{n=1}^{\infty} n^{k-1} |t_n - t_{n-1}|^k < \infty,$$

where

$$t_n = \sum_{j=0}^{n} t_{nj} s_j.$$

Flett [4] established the following inclusion theorem for $|C, \alpha|_k$. If the series $\sum_{n=0}^{\infty} a_n$ is summable $|C, \alpha|_k$, it is also summable for $|C, \alpha|_r$ for each $r \geq k \geq 1,\ \alpha > -1,\ \beta > \alpha + \frac{1}{k} - \frac{1}{r}$. Especially, a series $\sum_{n=0}^{\infty} a_n$ which is $|C, \alpha|_k$ summability is also $\left|C, \beta\right|_k$ summability for $k \geq 1,\ \beta > \alpha > -1$.

If one sets $\alpha = 0$, from the above inclusion result, we have

Theorem A. *Let $k \geq 1$, then $(C, \alpha) \in B(A_k)$ for $\alpha > -1$.*

Many authors have devoted themselves to generalize the results of Flett ([1], [2], [5], [6]). For example, the most recent works on this topic can be found in [5] and [6].

We first generalize the concept of the absolutely kth-power conservative to the following

Definition 1.1. *Let $\varphi(x)$ be a nonnegative function defined on $[0, \infty)$, $\{\alpha_n\}$, $\{\beta_n\}$, $\{\gamma_n\}$ and $\{\delta_n\}$ be nonnegative sequences. We say that a matrix*

$$T := \left(t_{nj}\right) \in B(\alpha_n, \beta_n; \gamma_n, \delta_n; \varphi),$$

if

$$\sum_{n=1}^{\infty} \alpha_n \varphi\left(\beta_n |s_n - s_{n-1}|\right) < \infty,$$

implies that

$$\sum_{n=1}^{\infty} \gamma_n \varphi\left(\delta_n |t_n - t_{n-1}|\right) < \infty.$$

If $\alpha_n = \gamma_n = n^{-1},\ \beta_n = \delta_n = n,\ \varphi(x) = x^k,\ k \geq 1$, then $B(\alpha_n, \beta_n; \gamma_n, \delta_n; \varphi)$ reduces to $B(A_k)$.

We will give a general result (Theorem 2.1) on the sufficient conditions for a matrix belongs to $B(\alpha_n, \beta_n; \gamma_n, \delta_n; \varphi)$ when φ is convex. As applications of the general result, we investigate the conservatives of Cesáro matrices and Riesz matrices (see Theorem 3.3-Theorem 3.5). Among them, Theorem 3.3 is an essential generalization of Theorem A in the case when $\alpha \geq 0$ (see remark after Theorem 3.3).

2. Main Result

Let $T := \left(t_{nj}\right)$ be a lower triangular matrix, $\lambda = \{\lambda_n\}$ be a positive sequence. Set

$$\widetilde{t_{ni}} := \begin{cases} \sum_{j=i}^{n} t_{nj} - \sum_{j=i}^{n-1} t_{n-1,j}, & 0 \leq i \leq n-1, \\ t_{nn}, & i = n, \end{cases}$$

$$\widetilde{T}_n(\lambda) := \sum_{i=0}^{n} \lambda_i \left|\widetilde{t_{ni}}\right|.$$

Theorem 2.1. *Let $\varphi(x)$ be a nonnegative convex function defined on $[0,\infty)$, $T := \left(t_{nj}\right)$ be a lower triangular matrix satisfying $\sum_{j=0}^{n} t_{nj} = 1$, and let $\{\alpha_n\}$ be a nonnegative sequence. If $\lambda = \{\lambda_n\}$ is a positive sequence such that* [1)]

$$\lambda_n^{-1} \sum_{j=n}^{\infty} \alpha_j \left|\widetilde{t_{jn}}\right| \left(\widetilde{T_j}\left(\lambda^{-1}\right)\right)^{-1} = O\left(A_n\right),\ n \geq 1, \tag{1}$$

then

$$T \in B\left(A_n, \lambda_n; \alpha_n, \left(\widetilde{T_n}\left(\lambda^{-1}\right)\right)^{-1}; \varphi\right). \tag{2}$$

Proof. Since (set $s_{-1} := 0$)

$$\begin{aligned} t_n &= \sum_{j=0}^{n} t_{nj} s_j = \sum_{j=0}^{n} t_{nj} \left(\sum_{i=0}^{j} (s_i - s_{i-1})\right) \\ &= \sum_{i=0}^{n} (s_i - s_{i-1}) \left(\sum_{j=i}^{n} t_{nj}\right), \end{aligned}$$

then

$$\begin{aligned} t_n - t_{n-1} &= \sum_{i=0}^{n} (s_i - s_{i-1}) \left(\sum_{j=i}^{n} t_{nj}\right) - \sum_{i=0}^{n-1} (s_i - s_{i-1}) \left(\sum_{j=i}^{n-1} t_{n-1,j}\right) \\ &= \sum_{i=0}^{n} \widetilde{t_{ni}} (s_i - s_{i-1}) = \sum_{i=1}^{n} \widetilde{t_{ni}} (s_i - s_{i-1}), \end{aligned}$$

where in the last inequality, we used the fact $\widetilde{t_{n0}} = 0$, which follows from $\sum_{j=0}^{n} t_{nj} = 1$ and the definition of $\widetilde{t_{n0}}$. Therefore,

$$\left(\widetilde{T_n}\left(\lambda^{-1}\right)\right)^{-1} |t_n - t_{n-1}| \leq \left(\widetilde{T_n}\left(\lambda^{-1}\right)\right)^{-1} \sum_{i=0}^{n} \lambda_i^{-1} \left|\widetilde{t_{ni}}\right| \left(\lambda_i |s_i - s_{i-1}|\right).$$

Since

$$\left(\widetilde{T_n}\left(\lambda^{-1}\right)\right)^{-1} \sum_{i=0}^{n} \lambda_i^{-1} \left|\widetilde{t_{ni}}\right| = 1,$$

by the well-known Jensen's inequality and (1), we get

$$\begin{aligned} &\sum_{n=1}^{\infty} \alpha_n \varphi\left(\left(\widetilde{T_n}\left(\lambda^{-1}\right)\right)^{-1} |t_n - t_{n-1}|\right) \\ &\leq \sum_{n=1}^{\infty} \alpha_n \varphi\left(\left(\widetilde{T_n}\left(\lambda^{-1}\right)\right)^{-1} \sum_{i=1}^{n} \lambda_i^{-1} \left|\widetilde{t_{ni}}\right| \left(\lambda_i |s_i - s_{i-1}|\right)\right) \\ &\leq \sum_{n=1}^{\infty} \alpha_n \left(\widetilde{T_n}\left(\lambda^{-1}\right)\right)^{-1} \sum_{i=1}^{n} \lambda_i^{-1} \left|\widetilde{t_{ni}}\right| \varphi\left(\lambda_i |s_i - s_{i-1}|\right) \\ &= \sum_{n=1}^{\infty} \varphi\left(\lambda_n |s_n - s_{n-1}|\right) \lambda_n^{-1} \sum_{j=n}^{\infty} \alpha_j \left|\widetilde{t_{jn}}\right| \left(\widetilde{T_j}\left(\lambda^{-1}\right)\right)^{-1} \\ &= O(1) \sum_{n=1}^{\infty} A_n \varphi\left(\lambda_n |s_n - s_{n-1}|\right), \end{aligned}$$

[1)]Denote by $\lambda^{-1} = \{\lambda_n^{-1}\}$.

which implies (2). □

3. Applications of The Main Result

Lemma 3.1 ([7]). *(i)A_n^α is positive for $\alpha > -1$, increasing (as a function of n) for $\alpha > 0$ and decreasing for $-1 < \alpha < 0$; and $A_n^0 = 1$ for all n.*

(ii) $A_n^\alpha \simeq \frac{n^\alpha}{\Gamma(\alpha+1)}$.

Lemma 3.2. *For any $\varepsilon > 0$, we have*

$$\sum_{n=i}^{\infty} \frac{A_{n-i}^{\alpha-1}}{n^\varepsilon A_n^\alpha} = O\left(i^{-\varepsilon}\right),\ \alpha \geq 0, \tag{3}$$

and

$$\sum_{n=i}^{\infty} \frac{\left|A_{n-i}^{\alpha-1}\right|}{n^\varepsilon A_n^\alpha} = O\left(i^{-\varepsilon-\alpha}\right),\ \alpha < 0. \tag{4}$$

Proof. When $\varepsilon > 0,\ \alpha \geq 0$, by Lemma 3.1, we get

$$\begin{aligned}\sum_{n=i}^{\infty} \frac{A_{n-i}^{\alpha-1}}{n^\varepsilon A_n^\alpha} &= O(1)\left(\frac{1}{i^\varepsilon A_i^\alpha}\sum_{n=i}^{2i} A_{n-i}^{\alpha-1} + \sum_{n=2i+1}^{\infty} \frac{A_{n-i}^{\alpha-1}}{n^\varepsilon A_n^\alpha}\right)\\ &= O(1)\left(\frac{1}{i^\varepsilon A_i^\alpha}\sum_{n=0}^{i} A_n^{\alpha-1} + \sum_{n=2i+1}^{\infty} \frac{(n-i)^{\alpha-1}}{n^{\varepsilon+\alpha}}\right)\\ &= O(1)\left(i^{-\varepsilon} + \sum_{n=2i+1}^{\infty} n^{-1-\varepsilon}\right)\\ &= O\left(i^{-\varepsilon}\right),\end{aligned}$$

which gives (3). When $\varepsilon > 0,\ \alpha < 0$, by Lemma 3.1, we get

$$\sum_{n=i+1}^{2i}\left|A_{n-i}^{\alpha-1}\right| = \left|\sum_{n=i+1}^{2i} A_{n-i}^{\alpha-1}\right| = \left|\sum_{n=0}^{i} A_n^{\alpha-1} - A_0^{\alpha-1}\right| = \left|A_i^\alpha - A_0^{\alpha-1}\right| = O(1)$$

and

$$\sum_{n=2i+1}^{\infty} \frac{\left|A_{n-i}^{\alpha-1}\right|}{n^\varepsilon A_n^\alpha} = O(1)\sum_{n=2i+1}^{\infty} \frac{(n-i)^{\alpha-1}}{n^{\varepsilon+\alpha}} = O(1)\sum_{n=2i+1}^{\infty} n^{-1-\varepsilon} = O\left(i^{-\varepsilon}\right),$$

Therefore, we also have (4). □

A non-negative sequence $\{a_n\}$ is said to be almost decreasing, if there is a positive constant K such that

$$a_n \geq K a_m$$

holds for all $n \leq m$, and it is said to be quasi-β−power increasing with some real number β, if $\left\{n^\beta a_n\right\}$ is almost decreasing.

Theorem 3.3. *Let $\varphi(x)$ be a nonnegative convex function defined on $[0,\infty)$.*
(A) If $\{\alpha_n\}$ is a nonnegative sequence such that $\{\alpha_n\}$ is quasi-$\varepsilon-$power decreasing for some $\varepsilon > 0$. Then

$$(C,\alpha) \in B(\alpha_n, n; \alpha_n, n; \varphi), \quad \alpha \geq 0.$$

(B) If $k \geq 1,\ \delta < \frac{1}{k},\ \gamma \in R$, then

$$(C,\alpha) \in B\left(n^{\delta k-1}\log^{\gamma} n, n; n^{\delta k-1}\log^{\gamma} n, n; \varphi\right), \quad \alpha \geq 0. \tag{5}$$

Proof. Let

$$t_{nj} := \frac{A^{\alpha-1}_{n-j}}{A^{\alpha}_n},\ j = 0, 1, \cdots, n;\ \alpha > -1.$$

Then, for $0 \leq i \leq n-1$,

$$\begin{aligned} \widetilde{t}_{ni} &= \frac{1}{A^{\alpha}_n}\sum_{j=i}^{n} A^{\alpha-1}_{n-j} - \frac{1}{A^{\alpha}_{n-1}}\sum_{j=i}^{n} A^{\alpha-1}_{n-1-j} \\ &= \frac{1}{A^{\alpha}_n}\sum_{j=0}^{n-i} A^{\alpha-1}_{j} - \frac{1}{A^{\alpha}_{n-1}}\sum_{j=0}^{n-1-i} A^{\alpha-1}_{j} \\ &= \frac{A^{\alpha}_{n-i}}{A^{\alpha}_n} - \frac{A^{\alpha}_{n-1-i}}{A^{\alpha}_{n-1}} = \frac{i}{n}\frac{A^{\alpha-1}_{n-i}}{A^{\alpha}_n}, \end{aligned} \tag{6}$$

and

$$\widetilde{t}_{nn} = \frac{A^{\alpha-1}_0}{A^{\alpha}_n} = \frac{1}{A^{\alpha}_n}. \tag{7}$$

Taking $\lambda_n = n,\ n \geq 1$, by (6) and (7), we have

$$\widetilde{T}_n\left(\lambda^{-1}\right) = \sum_{i=1}^{n} \lambda_i^{-1}\left|\widetilde{t}_{ni}\right| = \frac{1}{nA^{\alpha}_n}\sum_{i=1}^{n} A^{\alpha-1}_{n-i} - \frac{A^{\alpha-1}_n}{nA^{\alpha}_n} \simeq \frac{1}{n},\ n \geq 1. \tag{8}$$

By (8) and (3), we have

$$\begin{aligned} \lambda_n^{-1}\sum_{j=n}^{\infty}\alpha_j\left|\widetilde{t}_{jn}\right|\left(\widetilde{T}_j\left(\lambda^{-1}\right)\right)^{-1} &= \sum_{j=n}^{\infty} j\alpha_j\left(\frac{A^{\alpha-1}_{j-n}}{jA^{\alpha}_j}\right) \\ &= O\left(\sum_{j=n}^{\infty} j^{\varepsilon}\alpha_j\frac{A^{\alpha-1}_{n-j}}{j^{\varepsilon}A^{\alpha}_j}\right) \\ &= O\left(n^{\varepsilon}\alpha_n\sum_{j=n}^{\infty}\frac{A^{\alpha-1}_{n-j}}{j^{\varepsilon}A^{\alpha}_j}\right) \\ &= O(\alpha_n). \end{aligned}$$

Therefore, applying Theorem 2.1, we obtain (A).

Let $\alpha_n = n^{\delta k-1}\log^{\gamma} n,\ k \geq 1,\ \delta < \frac{1}{k}, \gamma \in R$. Since $\delta k - 1 < 0$, there is an $\varepsilon > 0$ such that $\varepsilon + \delta k - 1 < 0$, hence $\{n^{\varepsilon}\alpha_n\}$ is almost decreasing. Therefore, (B) follows from (A). □

Remark. Theorem A is (5) in the special case when $\delta = \gamma = 0$ and $\varphi(x) = x^k,\ k \geq 1$.

Theorem 3.4. *Let $\varphi(x)$ be a nonnegative convex function defined on $[0, \infty)$.*
(A) If $\{\alpha_n\}$ is a nonnegative sequence such that $\{n^\alpha \alpha_n\}$ is quasi-$\varepsilon-$power decreasing for some $\varepsilon > 0$. Then

$$(C, \alpha) \in B(\alpha_n, n; \alpha_n, n; \varphi), \quad -1 < \alpha < 0.$$

(B) If $k \geq 1,\ \delta < \frac{1-\alpha}{k},\ \gamma \in R$, then

$$(C, \alpha) \in B\left(n^{\delta k-1} \log^\gamma n, n; n^{\delta k-1} \log^\gamma n, n^{1+\alpha}; \varphi\right), \quad -1 < \alpha < 0. \tag{9}$$

Proof. When $-1 < \alpha < 0$, we have

$$\begin{aligned}
\widetilde{T}_n\left(\lambda^{-1}\right) &= \frac{1}{nA_n^\alpha}\sum_{i=1}^{n}\left|A_{n-i}^{\alpha-1}\right| = \frac{1}{nA_n^\alpha}\left|\sum_{i=1}^{n-1} A_{n-i}^{\alpha-1}\right| + \frac{A_0^{\alpha-1}}{nA_n^\alpha} \\
&= \frac{1}{nA_n^\alpha}\left|\sum_{i=0}^{n} A_{n-i}^{\alpha-1} - A_n^{\alpha-1} - A_0^{\alpha-1}\right| + \frac{1}{nA_n^\alpha} \\
&= \frac{1}{nA_n^\alpha}\left|A_n^\alpha - A_n^{\alpha-1} - A_0^{\alpha-1}\right| + \frac{1}{nA_n^\alpha} \\
&\geq C\frac{1}{nA_n^\alpha} \geq Cn^{-(1+\alpha)}.
\end{aligned} \tag{10}$$

By (6), (4), (10) and noting that $\{n^\alpha \alpha_n\}$ is quasi-$\varepsilon-$power decreasing with $\varepsilon > 0$, we have

$$\begin{aligned}
\lambda_n^{-1}\sum_{j=n}^{\infty}\alpha_j\left|\widetilde{t}_{jn}\right|\left(\widetilde{T}_j\left(\lambda^{-1}\right)\right)^{-1} &= O(1)\sum_{j=n}^{\infty} j^{1+\alpha}\alpha_j\left(\frac{\left|A_{j-n}^{\alpha-1}\right|}{jA_j^\alpha}\right) \\
&= O(1)\sum_{j=n}^{\infty} j^{\alpha+\varepsilon}\alpha_j\frac{\left|A_{j-n}^{\alpha-1}\right|}{j^\varepsilon A_j^\alpha} \\
&= O\left(n^{\alpha+\varepsilon}\alpha_n\sum_{j=n}^{\infty}\frac{\left|A_{j-n}^{\alpha-1}\right|}{j^\varepsilon A_j^\alpha}\right) \\
&= O(\alpha_n),
\end{aligned}$$

which together with Theorem A yields to (A).
(B) can be deduced from (A) directly. □

Theorem 3.5. *Let $\varphi(x)$ be a nonnegative convex function defined on $[0, \infty)$, $\{\alpha_n\}$ be a nonnegative sequence and $\lambda = \{\lambda_n\}$ be a positive sequence. Let $T = \left(t_{nj}\right)$ be a lower triangular matrix with the entries having the form $\frac{p_j}{P_n}$, where $p_j > 0$ for $0 \leq j \leq n$ and $P_n = \sum_{j=0}^{n} p_j$. If*

$$n\lambda_n^{-1}P_{n-1} = O\left(\sum_{i=1}^{n}\lambda_i^{-1}P_{i-1}\right), \tag{11}$$

and

$$\sum_{j=n}^{\infty}\frac{\alpha_j\lambda_j}{jP_{j-1}} = O\left(\frac{\alpha_n\lambda_n}{P_{n-1}}\right), \tag{12}$$

then

$$T \in B\left(\alpha_n, \lambda_n; \alpha_n, \frac{\lambda_n P_n}{np_n}; \varphi\right).$$

Proof. First, we have

$$\begin{aligned}\widetilde{t_{ni}} &= \sum_{j=i}^{n} t_{nj} - \sum_{j=i}^{n-1} t_{nj} \\ &= \frac{p_n}{P_n} + \left(\frac{1}{P_n} - \frac{1}{P_{n-1}}\right)\sum_{j=i}^{n-1} p_j \\ &= \frac{p_n}{P_n} - \frac{p_n}{P_n P_{n-1}}\left(P_{n-1} - P_{i-1}\right) \\ &= \frac{p_n P_{i-1}}{P_n P_{n-1}},\ 1 \le i \le n-1, \end{aligned} \tag{13}$$

and

$$\widetilde{t_{n0}} = 0,\ \widetilde{t_{nn}} = \frac{p_n}{P_n}. \tag{14}$$

By (11), we have

$$\left(\widetilde{T_n}\left(\lambda^{-1}\right)\right)^{-1} = \left(\frac{p_n}{P_n P_{n-1}} \sum_{i=1}^{n} \lambda_i^{-1} P_{i-1}\right)^{-1} = O\left(\frac{\lambda_n P_n}{np_n}\right). \tag{15}$$

By (12)-(14), we have

$$\begin{aligned}\lambda_n^{-1} \sum_{j=n}^{\infty} \alpha_j \left|\widetilde{t_{jn}}\right| \left(\widetilde{T_j}\left(\lambda^{-1}\right)\right)^{-1} &= O\left(\lambda_n^{-1} P_{n-1} \sum_{j=n}^{\infty} \frac{\alpha_j \lambda_j}{jP_{j-1}}\right) \\ &= O\left(\alpha_n\right). \end{aligned} \tag{16}$$

We obtain Theorem 3.5 by combining Theorem 2.1 with (15) and (16). □

Now, we give a special application of Theorem 3.5.
Let

$$p_0 = 1,\ p_n = n^{\alpha},\ n \ge 1, \alpha > -1,$$

$$\lambda_n = n,\ n \ge 1,$$

and

$$\alpha_n = n^{\delta k - 1},\ n \ge 1,\quad k > 0,\ \delta < \frac{1+\alpha}{k}.$$

Then

$$\sum_{i=1}^{n} \lambda_i^{-1} P_{i-1} \simeq \sum_{i=1}^{n} i^{\alpha} \simeq n^{\alpha+1} \simeq n \lambda_n^{-1} P_{n-1},$$

and (note that $\delta k - 2 - \alpha < -1$)

$$\sum_{j=n}^{\infty} \frac{\alpha_j \lambda_j}{j P_{j-1}} = O(1) \sum_{j=n}^{\infty} j^{\delta k - 2 - \alpha}$$
$$= O\left(n^{-\delta k - 1 - \alpha}\right)$$
$$= O\left(\frac{\alpha_n \lambda_n}{P_{n-1}}\right),$$

Therefore, Theorem 3.5 yields to

$$T \in B\left(n^{\delta k - 1}, n; n^{\delta k - 1}, n; \varphi\right).$$

In particular, taking $\delta = 0$, $\varepsilon = 1$, $k \geq 1$, we have $T \in B(A_k)$.

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